A method for automated in-field interface-wave inversion to estimate shallow subsurface-strength for offshore wind turbine construction

Introduction

With the ongoing push for offshore windmill construction, there is a need for fast yet accurate enough estimation of the shear-modulus in the near-surface below the ocean-bottom, as this modulus facilitates effective foundation design for offshore wind-parks. It is known that the dispersion of the phase- or group-velocity of interface waves such as Scholte or Stoneley waves that travel along the ocean floor can be inverted for the shear-wave velocity in the shallow subsurface underneath the ocean-floor. This shear-wave velocity then serves as a proxy for the shear-modulus and thus for the subsurface strength. Due to the nonlinear dependence of these velocities on the shear-wave velocity, these inversions are nonlinear and are typically solved in an iterative fashion (or using model search methods). While these methods can provide good results, they still need a reasonably accurate starting model to converge to a good solution.

We present an approximate method to invert the fundamental-mode phase-velocity dispersion of such waves that does not need a starting model, is accurate except in overly complex geological settings and requires minimal parameterization. Combined with its computational efficiency, the method is therefore a good candidate for automated and real-time inversion for the shear-wave velocity. Given these characteristics, we envision that this method in the future can be integrated into a seismic data-acquisition system that records the needed interface waves to allow real-time automated in-field estimation of the shear-wave velocity during acquisition. Having this ability would allow determination of optimal placement of expensive cone-penetration tests (CPT) immediately after data acquisition to obtain maximum added value of the CPTs. For example, CPTs could be placed only in areas where the interface-wave analysis we propose does not provide a high-enough quality estimate of the shallow shear-wave velocity. In this way the number of CPTs can potentially be minimized and the costs of the needed geotechnical campaign to build a ground model therefore reduced.

The most common method of inverting surface- or interface-waves for the shear-wave velocity is through doing local 1D inversions. This is the typical approach used in the well-known Multi-Channel Analysis of Surface-waves (MASW) method (e.g., Xia, 2014). Even though much more advanced methods such as elastic full waveform inversion (E-FWI) can also be used to invert surface-waves for the shear-wave velocity (e.g., Borisov *et al*., 2020), such an approach is currently likely to be an overkill to obtain a shear-wave velocity estimate that is sufficient for the purpose of foundation design for offshore windmill construction. We therefore (at least for now) focus our attention on a 1D method and envision it to be used in a standard MASW setting to build a two- or three-dimensional shear-wave velocity model through local one-dimensional inversions. We note that this approach can also be useful to create a starting model needed for E-FWI if in the future such an approach would be desirable.

Method and Theory

Recently Haney and Tsai (2015) presented a linear relation between the squared phase-velocity and the squared shear-wave interval velocities for Rayleigh and Love waves. Since this relation is analogous to the known Dix equation (Dix, 1955), which relates the squared stacking velocities in reflection seismic profiling to the squared interval velocities, they refer to this relation as a Dix-type equation. Haney and Tsai (2015) use this relation to setup a linear 1D inversion to get the interval shear-wave velocity in a layered model from the measured phase- or group-velocity dispersion. The main benefits of this method are that it requires no starting model and that it needs minimal parameterization. Moreover, this method has been shown to produce a reasonably accurate shear-wave velocity model, except in overly complex media. If in a complex geological setting a non-satisfactory model is obtained, the obtained model can be used as a good starting model for subsequent iterative methods in a nonlinear inversion.

We extended this method to the offshore setting to allow inversion of Scholte and Stoneley waves at sea. The analogous Dix-type equation to equation (27) from Haney and Tsai (2017) is given by

$$c_i^2 - \gamma_i = \sum_{j=1}^{N} M_{ij}\beta_j^2, \qquad (1)$$

where c_i is the measured phase-velocity for a given frequency f_i, β_j is the interval shear-wave velocity at depth z_j underneath the ocean floor, M_{ij} is the matrix relating the squared phase-velocities to the squared interval shear-wave velocities, and γ_i is a correction factor that depends mainly on c_i, f_i and the water-depth (H). Key in this method is that the matrix M_{ij} depends on the measured phase-velocities as well as an effective homogeneous shear-wave velocity β_i^{eff} that is different for each frequency f_i. This β_i^{eff} is found by simply solving the well-known fundamental-mode dispersion relation for Scholte waves for the value of β_i^{eff} given the measured phase velocity c_i for each frequency f_i.

Equation (1) can be used to setup a simple linear regularized least-squares inversion of the measured phase-velocities of the fundamental-mode interface waves for the interval shear-wave velocities β_j using the exact same procedure as used by Haney and Tsai (2017) with two minor adaptations. First, we replace the squared phase-velocities on the right-hand side of equation (28) in Haney and Tsai (2017), with $c_i^2 - \gamma_i$. Furthermore, for the a-priori model β_0 in that same equation (28), we use a new direct data-mapping method we developed to estimate an approximate a-priori model β_0. This method does not use any inversion. It is a direct mapping of the measured phase-velocities as a function of frequency to an approximate interval shear-wave velocity model as a function of depth. The thus-obtained a-priori model β_0 is used by the inversion to constrain the inversion in regions where the measured data has no resolution. Other than these two modifications the exact same procedure as used by Haney and Tsai (2017) can be followed to do the inversion. Because Scholte and Stoneley waves are mainly sensitive to the S-wave velocity and depend only weakly to the P-wave velocity, the method assumes an estimate of the Poisson's ratio (ν) to be known a-priori, like most surface-wave inversion methods. Beside this, the only parameters needed to do the inversion are the water depth (H) and an estimate of the level of noise in the picked fundamental-mode dispersion curve. Since no other parameters are needed and since these parameters can be safely assumed to be known, the method is a good candidate for a full automation.

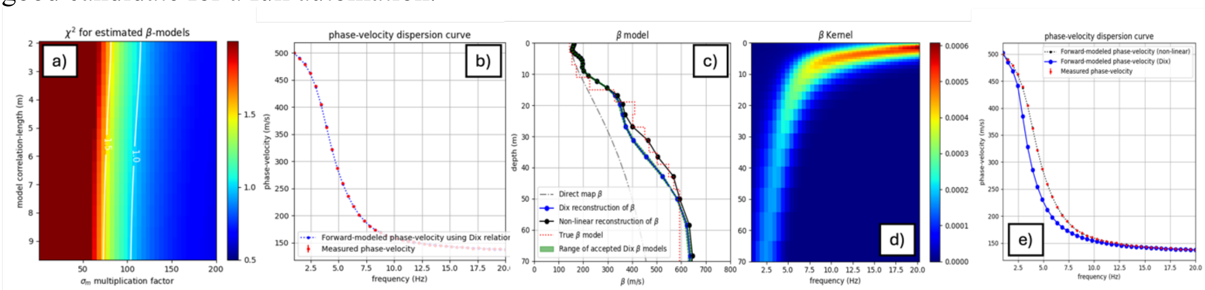

Figure 1. *a) grid search for regularization parameters to find the models that fit the data with 1.0 <* χ^2 *< 1.5 with b) the synthetic phase-velocity measurements indicated by the red dots; c) the* $\beta(z)$ *models obtained using the direct mapping of the phase-velocities (grey dashed-dotted line), the linear Dix-type inversion[blue line,* $\beta_{Dix}(z)$*] and the non-linear inversion (the black line); d) sensitivity kernel [i.e. matrix M in equation (1)] for the Dix-type inversion showing good resolution up to about 25m depth; e) exact forward-modelled dispersion curve from the* $\beta_{Dix}(z)$ *model (in blue) shows a difference with the synthetic phase-velocity measurements, indicating the non-linear inversion can still generate an improved model beyond* $\beta_{Dix}(z)$ *(as can be seen in subfigure c).*

We emphasize that the method from Haney and Tsai (2017) uses two regularization parameters. First it uses a correlation length (or smoothing distance) to force a smoothness constraint on the inverted models [see the definition of the model covariance matrix in equation (24) of Haney and Tsai (2017)]. Secondly, a model standard deviation is used in the model covariance matrix. This is typically set as a multiple of the estimated data standard deviation. The correlation length and this data-covariance multiplication factor are the two regularization parameters used in the method. Because of the computational efficiency of the linear inversion, these can be found through a simple grid search. The inversion procedure therefore does many inversions for many combinations of the regularization parameters, and all models that fit the data between one and one-and-a-half standard deviations (i.e. $1.0 < \chi^2 < 1.5$) are simply accepted. In this way overfitting of the data is avoided. An example of such a grid search in shown in Figure 1a. These calculations, and therefore the whole inversion, take about 1 sec on a modern laptop and can be further optimized using parallelization techniques if needed.

Results of testing with synthetic data

To show the accuracy of the method, Figure 1 shows the results of using the method to invert synthetically modelled fundamental-mode phase-velocity dispersion curves for a model relevant for some areas in the North-Sea. The true $\beta(z)$ model is indicated by the dotted red line in Figure 1c while the phase-velocities are shown by the red dots in Figure 1b and were modelled for frequencies up to 20 Hz. First, we obtain the a-priori shear-wave velocity model $\beta_0(z)$ through the direct mapping procedure we developed (the grey dashed-dotted line in Figure 1c). To calculate this model, besides the measured phase-velocities as a function of frequency $c_i(f_i)$, only H and ν need to be provided (see also Figure 3). We see that this model provides a reasonable first estimate of $\beta(z)$ and is rather accurate for the very shallow parts. The calculation of this model is instantaneous as it requires a direct mapping only.

Using this model as the a-priori $\beta_0(z)$ model in the linear Dix-type inversion, we obtain the $\beta(z)$ model indicated by the blue line in Figure 1c. Note that the accuracy of the model for depths larger than 10m and up to about 25m is particularly improved. The only extra needed input here is an estimate of the noise in the measurements $c_i(f_i)$ (see also Figure 3). Note that the matrix M_{ij} shown in Figure 1d indicates that the provided measurements result in resolution at depths between about 2m and 25m. We stress that the blue line really depicts an average of an ensemble of models (resulting from the grid search) that all fit the data to within one and one-and-a-half standard deviation (see Figure 1a).

Using the model obtained from the linear Dix-type inversion [$\beta_{Dix}(z)$], we can use an exact forward modelling of the phase-velocities using this model. Due to the approximate nature of the Dix-type forward modeling, there remains a slight mismatch between the measurements $c_i(f_i)$ and the predicted phase-velocities for small intermediate frequency range. This mismatch can be used in a subsequent non-linear inversion where the model obtained from the linear Dix-type inversion is used as a starting model. Doing this we obtained the $\beta(z)$ model indicated by the black line in Figure 1c. We note that this model is a slight improvement over the $\beta_{Dix}(z)$ model, in particular at intermediate depths from 25-50m.

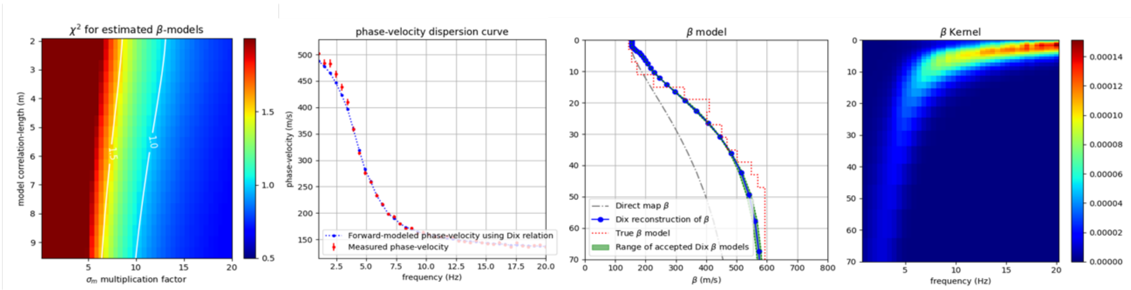

Figure 2. *Example of linear Dix-type inversion with noisy phase-velocity measurements.*

To indicate that the linear Dix-type inversion is stable in the presence of noise, Figure 2 shows an example of the inversion where uncorrelated noise was added to the phase-velocity measurements. This example shows that the method still produces a $\beta(z)$ model that is close to the true model, indicating the method is stable in the presence of noise.

Figure 3 shows a flow chart showing a possible use of the presented linear Dix-type inversion method, with accuracy increasing from the left to the right. The direct mapping of the phase-velocities to obtain a $\beta(z)$ model can be used as input to the linear Dix-type inversion or as a final result. Similarly, the model obtained from the Dix-type inversion can be used as input (i.e. as a starting model) to a non-linear inversion or as a final result. This depends on the level of accuracy needed for the civil engineering task at hand. Note that with the current workflow with the newly developed linear Dix-type inversion, a full non-linear inversion also only needs H, ν and an estimate of the noise in the measurements as inputs. This means that the full non-linear inversion can also be fully automated.

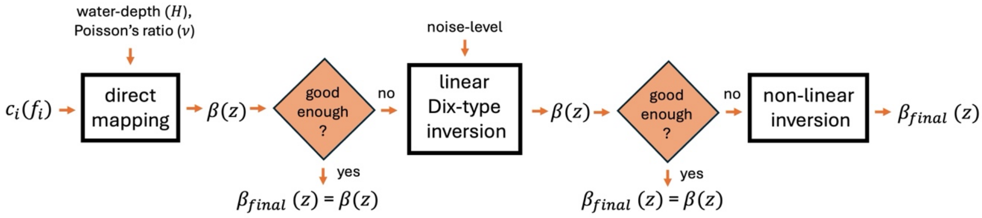

Figure 3. *Flow-chart indicating that the only inputs necessary to obtain a model from the linear Dix-type inversion or the non-linear inversion, are the water-depth (H), an estimate of the Poisson's ratio (ν) and the level of noise in the measured dispersion. Dependent on the level of accuracy needed in the model, the estimation of $\beta(z)$ can be halted after the direct mapping of the phase-velocities, the linear Dix-type inversion, or the non-linear inversion.*

Future work

We are working to incorporate the newly developed 1D Dix-type fundamental-mode phase-velocity inversion into an MASW procedure to be able to produce 2D or 3D models of the shear-wave velocity in the shallow subsurface underneath the ocean floor. Furthermore, combining the method with an automated dispersion-curve picker will facilitate testing the method in a realistic setting with field data.

Conclusions

The presented linear Dix-type inversion method does not need a starting model and needs only minimal parameters. The main parameters that are needed are the water-depth and an estimate of the noise in the measured dispersion curves. As such there are two ways of using it. It can be used to provide a rather accurate model in not overly complex subsurface settings. As such, in geotechnical and civil engineering settings in the context of offshore windturbine construction, the obtained inverted shear-wave velocity is likely good enough for use in foundation design. In more complex settings, or where more resolution is desired, it can also be used to obtain an initial model for more advanced inversion methods. In this context, it could serve as a way to obtain an initial model for the shear-wave velocity in elastic full waveform inversion. Because of the simplicity and computational efficiency of the method as well as the minimal need for input parameters, it is a good candidate for real-time in-field usage during acquisition and integration into the acquisition equipment itself.

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