Deep-learning-based uncertainty quantification for post-stack UHR seismic inversion

Quantitative interpretation of UHR data

For the characterization of the shallow subsurface for offshore wind farms, geotechnical data e.g. cone penetration testing (CPT) and ultra-high-resolution (UHR) seismic data are typically acquired. In this context, seismic methods are, however, typically relegated to structural characterization and shallow-hazard localization, which ignores their potential to infer quantitative properties of the subsurface. Seismic data are efficient to acquire and can ideally complement CPT measurements, which are highly local in nature (and relatively costly).

Quantitative inversion of seismic data requires dedicated processing workflows that have become commonplace for exploration applications but are not routinely applied for windfarm site characterization. For instance, these include source de-ghosting and de-signature and receiver-side de-ghosting. These can be challenging to apply for UHR data when accurate measurements are not available and because the magnitude of perturbations due to sea-surface waves cannot be ignored (Provenzano et al., 2020; Ramani et al., 2022; Henderson et al., 2023; Telling et al., 2024).

In this work, we aim to advance the *quantitative interpretation* of UHR data, by leveraging the de-ghosting workflow described in Telling et al., 2024, as a starting point. Here, we focus on investigating the *uncertainty* related to the acoustic impedance inversion from migrated stacks. This is particularly important in scenarios with limited data, such as windfarm site characterization, which rely on conservative estimates for, e.g., hazard detection. We explore the role of Bayesian uncertainty quantification with the machine-learning-based techniques discussed in Rizzuti and Vasconcelos, 2024. The goal is to demonstrate the feasibility and effectiveness of such techniques enabled by recent advances in deep learning.

Bayesian uncertainty quantification with multiscale invertible networks

In a post-stack acoustic seismic inversion, the data *likelihood* of $\boldsymbol{y}$ (seismic post-stack data) given $\boldsymbol{x}$ (logarithm of the acoustic impedance) is often assumed to be a normal distribution where the mean is given by a time derivative $\boldsymbol{D}_t$ followed by wavelet convolution $\boldsymbol{W}$ (fixed, for our purposes) applied to $\boldsymbol{x}$ (Izzatullah et al., 2023). Its log-probability is

$$-\log p(\boldsymbol{y}|\boldsymbol{x}) = \frac{1}{2\sigma^2}\|\boldsymbol{A}\boldsymbol{x}-\boldsymbol{y}\|^2 + \cdots, \qquad \boldsymbol{A} = \boldsymbol{W}\boldsymbol{D}_t.$$

Note that additional terms not dependent on $\boldsymbol{x}$ are indicated by "⋯". We assume that the *prior* distribution is differentiable and available in analytical form, for example:

$$-\log p(\boldsymbol{x}) = \frac{1}{2\tau_1^2}\|\boldsymbol{x}-\boldsymbol{x}_0\|^2 + \frac{1}{2\tau_2^2}\|\boldsymbol{B}\boldsymbol{x}\|^2 + \cdots.$$

Here, $\boldsymbol{x}_0$ is a background model. The weighting matrix $\boldsymbol{B}$, for instance, might promote models that are structurally aligned with some reference image. According to the Bayes' rule, the *posterior* distribution of $\boldsymbol{x}$ knowing $\boldsymbol{y}$ is

$$p(\boldsymbol{x}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x})p(\boldsymbol{x}).$$

The aim of this paper is a comprehensive characterization of the posterior distribution for some given data $\boldsymbol{y}$. The desired solution consists of generating random samples $\boldsymbol{x}\sim p(\boldsymbol{x}|\boldsymbol{y})$, from which any statistical moment can then be approximated.

In Rizzuti and Vasconcelos, 2024, a variational inference framework is presented where the posterior distribution is implicitly represented by a generative model $p(\boldsymbol{x}|\boldsymbol{y}) \approx p_{\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{y})$. The generative model is based on a special class of neural networks uniquely suited for uncertainty quantification and widely applied to seismic problems: normalizing flows (Kobyzev et al., 2021). In practice, samples from the target posterior distribution are generated with a transport map $T_{\boldsymbol{\theta}}$ by evaluating normally distributed random inputs $\boldsymbol{z}\sim p(\boldsymbol{z})$, e.g. $\boldsymbol{x}\sim p_{\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{y}) \Leftrightarrow \boldsymbol{x} = T_{\boldsymbol{\theta}}(\boldsymbol{z})$. The training is carried out by minimizing the Kullback-Leibler divergence:

$$\min_{\theta} \mathrm{KL}(p_{\theta}(\boldsymbol{x}) \| \, p(\boldsymbol{x}|\boldsymbol{y})) = E_{\boldsymbol{z}\sim p(\boldsymbol{z})} - \log p(T_{\theta}(\boldsymbol{z})|\boldsymbol{y}) + \log p_{\theta}\big(T_{\theta}(\boldsymbol{z})\big) + \cdots.$$

The advantage of variational inference approaches compared to more traditional Monte Carlo Markov chain methods lies in much more favorable computational scaling with respect to problem size (Zhang and Curtis, 2021).

An important feature of our approach is the ability to estimate the posterior distribution sequentially from coarse low-dimensional grids to fine grids, through so-called multiscale wavelet normalizing flows (Yu et al., 2020), as an effective way to manage the ill-conditioning of the inverse problem in object.

Example

We analyze field data acquired in the German North Sea for offshore windfarm development and focus on a small portion of a 2D line situated around the projected location of a CPT well-log situated approximately 160 m away from the seismic line. More technical details about data acquisition and processing, especially for source and receiver de-ghosting, are described in Telling et al., 2024.

The background acoustic impedance model is directly obtained from the migration interval velocity and Gardner's relation (see Figure 1). We inferred a zero-phase wavelet by fitting the background acoustic impedance to the post-stack trace corresponding to the well-log (projected) location (Figure 2).

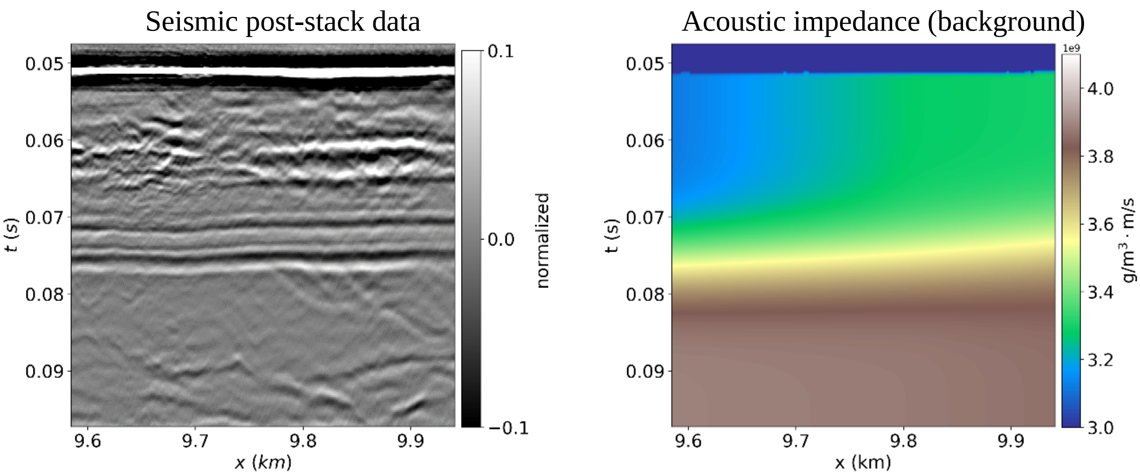

Figure 1 *Seismic post-stack data and background acoustic impedance. The background model was obtained from migration velocities together with Gardner's relation.*

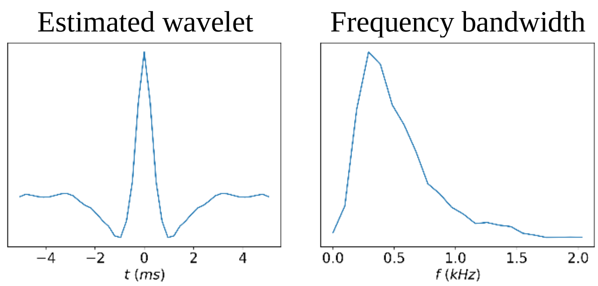

Figure 2 *Wavelet estimated by matching the background model to data.*

We depict a summary of the uncertainty quantification results in Figure 2. We show the point-wise conditional mean and standard deviation computed from the samples obtained by the trained generative model based on multiscale normalizing flows. Note that higher-order statistics can be produced just as easily.

In Figure 3, we highlight the multiscale behavior of the estimated posterior distribution. We show how the conditional mean and standard deviation results evolve by progressively adding finer resolution scales, from coarse to fine grids.

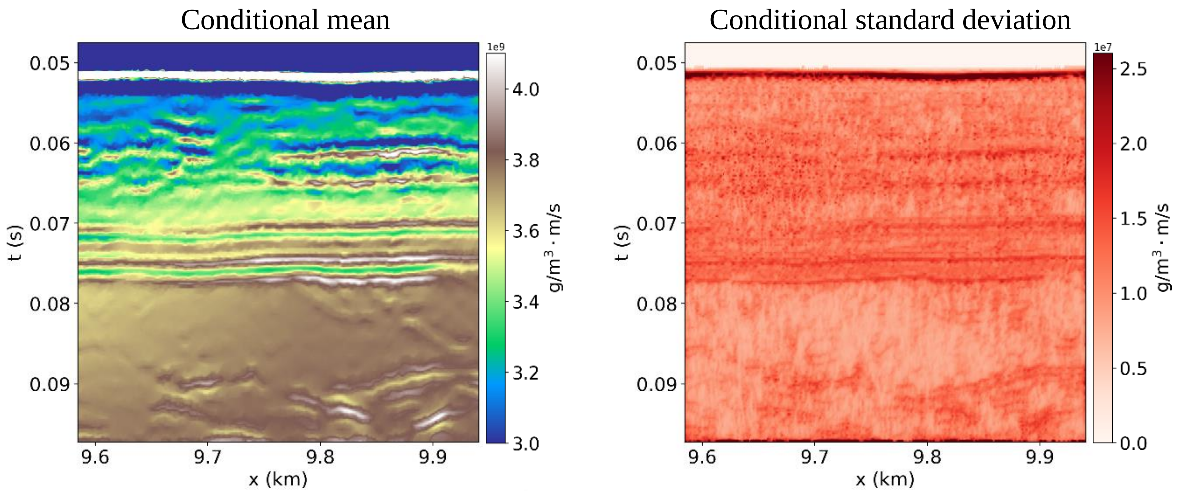

Figure 2 *Uncertainty quantification results obtained with the proposed method: acoustic impedance point-wise conditional mean and its standard deviation.*

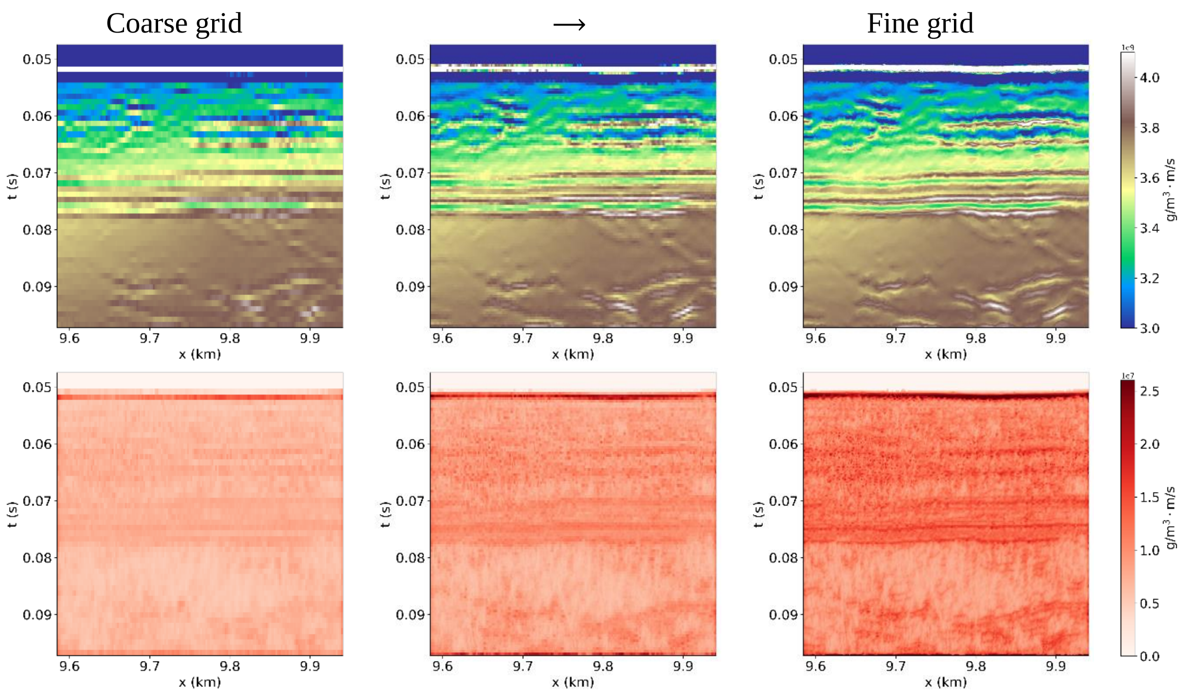

Figure 3 *Multiscale progression of the proposed uncertainty quantification framework, from coarse to fine grids*

The road ahead for probabilistic UHR quantitative inversion

The deep-learning workflow here proposed for the characterization of the uncertainty quantification for UHR post-stack data is a flexible and computationally feasible tool that has the potential to guide the decision-making process in offshore windfarm development in the future. While the example shown in this work only considers a 2D seismic line, the method can scale to 3D post-stack inversion problems

(Rizzuti and Vasconcelos, 2024). Although currently, UHR data is almost exclusively acquired in 2D, the proposed technology is already poised for future 3D acquisition developments.

One fundamental challenge for quantitative inversion for UHR data is the integration of CPT or seismic-CPT well-log information, especially for wavelet calibration and background acoustic impedance estimation. For example, well-log data contained in seismic-CPT measurements are sampled at a lower spatial rate than seismic data and the geo-mechanical properties such as density, are inferred only indirectly. Moreover, we expect the seismic data characteristics and local measurements to change abruptly even within tens of meters from the reference 2D line.

Going forward, we aim to enhance the quantitative characterization of UHR data by including seismic-CPT well-logs and wavelet effects in the uncertainty analysis. The method can be easily generalized to elastic impedance inversion, as well. Finally, another important aspect is the practical use of the inferred posterior probability with the integration of decision theory in the proposed Bayesian framework (Arnold and Curtis, 2018).

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